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A novel forecasting method based on multi-order fuzzy time series and technical analysis

Furong Ye[a], Liming Zhang[b], Defu Zhang[a,*], Hamido Fujita[c], Zhiguo Gong[b]

[a] *Department of Computer Science, Xiamen University, Xiamen, 361005, China*
[b] *Department of Computer and Information Science, University of Macau, Macau, China*
[c] *Faculty of Software and Information Science, Iwate Prefectural University, Iwate, Japan*

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ABSTRACT

Financial trading is one of the most common risk investment actions in the modern economic environment because financial market systems are complex non-linear dynamic systems. It is a challenge to develop the inherent rules using the traditional time series prediction technique. In this paper, we proposed a new forecasting method based on multi-order fuzzy time series, technical analysis, and a genetic algorithm. Multi-order fuzzy time series (first-order, second-order and third-order) are applied in the proposed algorithm, and to improve the performance, genetic algorithm is used to find a good domain partition. Technical analysis such as the Rate of Change (ROC), Moving Average Convergence/Divergence (MACD), and Stochastic Oscillator (KDJ) are introduced to construct multi-variable fuzzy time series, and exponential smoothing is used to eliminate noise in the time series. In addition to the root mean square error and mean square error, the directional accuracy rate (DAR) is also used in our empirical studies. We apply the proposed method to forecast five well-known stock indexes and the NTD/USD exchange rates. Experimental results demonstrate that our proposed method outperforms other existing models based on fuzzy time series.

1. Introduction

Fuzzy intervals are widely regarded as the fundamental problem for modeling fuzzy time series and are essential for model calculation and trend prediction [1]. As a result, fuzzy intervals are often taken as a key research problem in data analysis. Since Song and Chissom [1] introduced the concepts of fuzzy time series, such models have received much attention from researchers, and considerable research progress has been made afterwards. We can group the existing works into the following four categories according to the technique used to partition fuzzy intervals as follows.

As the first category, the works by Song [1,2], Chen [3], Hwang [4] and Lee [5] are regarded as the pioneer research efforts in this area. In their models, the minimum and maximum values of the sample data were rounded upward and downward, respectively, to determine the universe classification. Then, based on the size of the universe, they took an integer as the length of the interval to uniformly divide the universe.

The representative scholars of the second category include Huarng [6], Teoh [7], Jilani [8], and Yu [9]. They proposed to divide the interval based on the distribution of the sample data. The techniques used include adjusting the interval lengths

* Corresponding author. Tel.: +86 5925918207; fax: +86 5922580035.
E-mail address: dfzhang@xmu.edu.cn (D. Zhang).

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according to the density of the samples, defining a new distance formula and dividing intervals according to the distance distribution between samples, and determining the number of intervals according to the statistical peak of the samples.

Aladag [10,11], Yolcu [12] and Egrioglu [13,14], as the representatives of the third category, proposed to find the partition method of the optimal fuzzy subset by using optimization algorithms. Aladag [10] used neural networks to forecast in high order fuzzy time series, Yolcun [12] focused on a sing-variable constraint and proposed an efficient approach to identify the intervals' length, and the method of Egrioglu is based on SARIMA. Chen and Cheng's genetic algorithm [15–17] also falls in this category. The basic idea of methods is to use the prediction error as the objective function and seek for the minimum value of the objective function according to a certain step length. The interval with the minimum objective value is taken as the final partition of the model.

The fourth category of methods includes newly proposed clustering algorithms [18–21] whose idea is similar to the fuzzy clustering method (FCM) by Li [22]. The basic idea of these methods is to use an appropriate algorithm to perform cluster analysis on the sample data, and then determine the partition of each subinterval according to the clustering results.

Many investors often use technical indicators to analyze the stock market and predict its future trend [23]. Stevenson and John [24] applied a new technique year percentage change replacing enrollments as the universe of discourse. Multivariable fuzzy time series or multi-factor fuzzy time series based on technical indicators is used to solve the problem of prediction. Lee et al. [25] developed more techniques on prediction, which considered more factors and high-order to achieve better results. To avoid complicated matrix computations, Huarng et al. [26] handled forecasting problem with a multivariate heuristic model. To forecast the TAIEX, Yu and Huarng [27,28] proposed a bivariate model by using neural networks. Chen and Chang [29] handled fuzzy rules by clustering algorithms and assigned different weights to clusters. Chen and Chen [30] investigated TAIEX forecasting with fuzzy time series by considering main factor and secondary factor. Chen et al. [31] used a particle swarm optimization algorithm to improve their research. Kim et al. [32] used technical indicators to construct multiple classifiers for predicting a stock price index. Egrioglu et al. [33] predicted Belgian traffic accident casualties from 1974 to 2004 and used a feedforward neural network to deal with fuzzy relationships in bivariate time series. Avazbeigi et al. [34] applied three variable fuzzy time series for prediction of automobile production in Iran; in addition, tabu search is presented in their work. Park et al. [35] developed bivariate fuzzy time series, which underlying price is used as the second variable, to predict TAIEX and South Korea's KOSPI 200 index.

Data processing is an important step of data mining to improve the performance of data mining algorithms. In general, data mining algorithms fail to extract nonlinear valuable patterns from noisy data; therefore, many data smoothing methods are used to address this task. By "averaging out" the noise, they can extract nonlinear relations from the time series. For example, Zhang *et al.* [36] used nonparametric kernel regression to filter the noise in the time series. This paper preprocesses the training data using the exponential smoothing method [37] to obtain smooth training data values, but do not perform any processing on the testing data.

As the global financial markets are becoming deregulated, the modeling and forecast of financial market system are becoming more complex in the risk management and derivatives rating. However, one of the key aspects of complex statistical model in financial market is accurate forecasting that could yield significant profits and could also decrease investment risks [38]. Considering the stock prediction, the most frequently used forecasting methods are nonlinear models, for example, neural network [36,39], Markov modeling [40], genetic algorithm [41], fuzzy logic [42], support vector machine [43] and hybrid models [44]. However, fuzzy time series method has been regarded as one of important novel methods in this area. Thus far, there has been various research of handling stock index forecasting using fuzzy time series [45–49].

For fuzzy time series forecasting, Chen et al. [15,16,18,19,29–31,49], Yu and Huarng [4,9,26–28,39,45] did a lot of excellent works. Recently, Wei et al. [50] forecasted the trend of TAIEX stock by combining a linear model and moving average technical index. Chen and Chen [51] introduced binning-based partition and entropy-based discretization and proposed a new fuzzy time series model based on granular computing. Chen and Chen [52] proposed a new fuzzy forecasting model, replacing the fuzzy logical relationship groups with fuzzy-trend logical relationship groups and introducing the probabilities of trends. Cai et al. [53] used ant colony optimization to obtain a good partition of the universe of discourse, and auto-regression was introduced to better use historical information. These algorithms show good ability of achieving good forecasting results.

In this paper, a new hybrid multi-order fuzzy time series model is proposed for financial forecasting, and genetic algorithm is applied to obtain good partitions of the universe of discourse. Generally, only first-order fuzzy time series is used in this area, which we think is not appropriate. Generally, stock price is influenced by historical data. The price of a certain stock on a certain day is not only related to the price of the day before but also related to the price of the near past, although they might not have the same impact strength. Obviously first-order fuzzy time series neglects the influence from the price of the near past, and this may cause the inaccuracy of forecast. To establish contact between the predicted day and the near past days, a hybrid multi-order fuzzy time series is applied. Specifically, we extract first-order, second-order and third-order fuzzy time series and average the three fuzzy values to obtain the final predicted value. We stop at the third-order because the effect of mixing higher order fuzzy time series to the results is negligible, and it affects the computation speed. Additionally, higher orders need more time series data to train a forecasting model. After many experiments, better forecasting results are achieved by using uniform weights across the orders instead of different weights. The reason in using a genetic algorithm is that the operators in the algorithm like selection, crossover, and mutation, can help the model find excellent domain partition iteratively. In particular, we introduce three indicators, the Rate of Change (ROC), Moving Average Convergence/Divergence (MACD), and Stochastic Oscillator (KDJ), to construct the multi-variables fuzzy time series, where the main variable is ROC, and MACD and KDJ are the secondary variables. According to the experimental results, the proposed method is superior to other existing models based on fuzzy time series.

The proposed method is different from our previous methods [47] and [53]. FTSGA in [47] combines fuzzy time series and a genetic algorithm for stock forecasting and uses only first-order model and a single variable. ACO-AR in [53] used fuzzy time series and an ant colony algorithm and auto-regression for forecasting, and it is a multi-order and single variable model. A few other researchers concentrated on multi-variables and a multi-order fuzzy time series model, such as [54]. In those papers, variables are used dependently; therefore the whole method is like a summary of several single variable models. The proposed method in this paper is a multi-variable and multi-order fuzzy time series model, and it uses technical analysis to further improve the results.

To the best of our knowledge, the directional accuracy rate (DAR) of forecasting model is seldom considered in other forecasting models. In fact, the directional accuracy of forecasting model is more taken into account the trading strategy for investment than the root mean square error (RMSE) [36]. In addition, most forecasting models based on fuzzy time series test their results with only one stock index, and this is not sufficient to compare the generalization abilities of different models. This paper considers the root mean square error (RMSE) and the directional accuracy rate (DAR) as the evaluation standard simultaneously and calculates RMSE and DAR for several well-known stock indexes such as Dow Jones, NASDAQ, HSI, and SP500.

The rest of this paper is organized as follows. The basic concepts of fuzzy time series are introduced in Section II, and technical analysis is briefly introduced in Section III. In Section IV, we present a novel framework for financial forecasting. In Section V, we develop a new forecasting method based on multi-order, Genetic algorithm and technical indicators. In Section VI, the forecasting results of the proposed method are compared with those of the existing models for forecasting six well-known financial time series. The conclusions are given in Section VII.

2. Fuzzy time series

The fuzzy time series was proposed by Song and Chissom in 1993 [1]. Up to now, fuzzy time series have been successfully used to solve many practical problems. To comprehend and apply fuzzy time series, we give the introduction of basic concepts of fuzzy time series as follows.

$U=\{u_1, u_2, \dots, u_n\}$ denotes the universe of discourse. A fuzzy set A in U is given as follows

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdots + f_A(u_n)/u_n \tag{1}$$

where $f_A : U \to [0,1]$ is the membership function of the fuzzy set A, and $f_A(u_i)$ is the membership degree of u_i belonging to A, and $f_A(u_i) \in [0,1]$ $(1 \le i \le n)$.

Definition 2.1. Let $Y(t)$ $(t=0, 1, \dots)$ denote a subset of real numbers R and the universe of discourse. Fuzzy sets $f_i(t)$ $(i=1, 2, \dots)$ are defined on $Y(t)$, and assume that $F(t)$ is a collection of $f_i(t)$ $(i=1, 2, \dots)$. Then, $F(t)$ is called a fuzzy time series on $Y(t)$ $(t=0, 1, \dots)$.

Definition 2.2. Assume that $R(t, t-1)$ is the fuzzy relation between $F(t-1)$ and $F(t)$ and both $F(t-1)$ and $F(t)$ are fuzzy sets. The relationship can be shown as $F(t)=F(t-1)oR(t, t-1)$, then $F(t)$ is called derived from $F(t-1)$, namely, $F(t-1) \to F(t)$, where the symbol "o" is the max-min composition operator. $F(t)=F(t-1)oR(t, t-1)$ is called the first-order model of $F(t)$.

Definition 2.3. Assume that $F(t-1)=A_i$ and $F(t)=A_j$, then $A_i \to A_j$ represents the fuzzy logic relationship between $F(t-1)$ and $F(t)$, where A_i(the left-hand side of $A_i \to A_j$) is called the current state of the fuzzy logic relationship (FLR) and A_j (the right-hand side of $A_i \to A_j$) is called the next state of the fuzzy logic relationship (FLR).

Definition 2.4. .$F(t)$ denotes a fuzzy time series, if $F(t)$ is caused by $F(t-1), F(t-2), \dots, F(t\text{-}n)$, then $F(t)$ is n-order fuzzy time series. For example, in this paper the expression of first-order fuzzy time series is $F(t-1) \to F(t)$; for the second-order, it is $F(t-2), F(t-1) \to F(t)$; and for the third–order, it is $F(t-3), F(t-2), F(t-1) \to F(t)$.

Definition 2.5. Let $R(t, t-1)$ be the fuzzy relation defined on $F(t-1)$ and $F(t)$. If $R(t, t-1)=R(t-1, t-2)$ for any t, then $F(t)$ is called a time invariant fuzzy time series. Otherwise, it is called a time variant fuzzy time series.

Definition 2.6. Let $F(t)$ be a fuzzy time series, if $F(t)$ is caused by $((F_1(t-1), F_1(t-2),\dots,F_1(t-n)), (F_2(t-1), F_2(t-2),\dots,F_2(t-n)),\dots, (F_m(t-1), F_m(t-2),\dots,F_m(t-n)))$, then it is called m-variate n-order fuzzy time series. For example, 4-variate 3-order fuzzy logic relationship is as follows:

$$\{(F_1(t-1), F_1(t-2), F_1(t-3)), (F_2(t-1), F_2(t-2), F_2(t-3)), (F_3(t-1), F_3(t-2), F_3(t-3)), \\ (F_4(t-1), F_4(t-2), F_4(t-3))\} \to F(t)$$

3. Technical analysis

Technical analysis is considered as a good analysis methodology in finance, it predicts the direction of prices by using historical information. Some indicators that obtain patterns through trading volume and price and describe behavior of relative prices have become the most important tools of technical analysis. The well-known indicators include the relative moving averages, strength index, regressions, business cycles, classically, recognition of chart patterns. Basic explanations for these financial terms can be referred to [23]. According to many experiments, ROC, MACD and KDJ are selected as variables for the model in this paper.

The Rate of Change (ROC) reflects the percentage difference between the closing price N days ago and today's closing price. It fluctuates around zero. The ROC rises as the price increases, and when the price decreases, the ROC drops. If the price experiences a sharp change, the ROC responses a great change.

The ROC can be seen as a momentum technical indicator, which identifies high lows and zero line crossovers. The ROC reflects the sales and purchase level of markets. The higher ROC reflects a more overbought security or vice versa. However, in many cases, the extremely overbought/oversold ROC may indicate a continuation of the recent trend.

This paper sets N = 1, and ROC of the *t*th day is computed as follows:

$$ROC_t = \frac{Close_t - Close_{t-1}}{Close_{t-1}} \times 100\% \tag{2}$$

MACD (Moving Average Convergence/Divergence) can help to reveal changes of stock prices and present a trend in a period of time. MACD is a set of three time series calculated from historical price data, and it is common to choose the closing price. These three series are: the MACD series proper, the "signal" or "average" series, and the "divergence" series which is the difference between the two. The MACD series is the difference between a "fast" (short period) exponential moving average (EMA), and a "slow" (longer period) EMA of the price series. The average series is an EMA of the MACD series itself.

The notation "MACD(a, b, c) " usually denotes the indicator where the MACD series is the difference of EMAs with characteristic times a and b, and the average series is an EMA of the MACD series with characteristic time c. These parameters are usually measured in days. MACD(12,26,9) is with most commonly selected parameters. For example, MACD(12,26, 9) is computed as follows:
EMA of 12 days

$$EMA_{12} = \frac{11}{13} \times EMA_{12} \text{ of last day} + \frac{2}{13} \times \text{the closing price of today}$$

EMA of 26 days

$$EMA_{26} = \frac{25}{27} \times EMA_{26} \text{ of last day} + \frac{2}{27} \times \text{the closing price of today}$$

DIF: $\text{DIF} = EMA_{12} - EMA_{26}$

$$\text{DEA of today} = \frac{8}{10} \times \text{DEA of last day} + \frac{2}{10} \times \text{DIF of today}$$

$$\text{MACD} = (\text{DIF} - \text{DEA}) \times 2$$

Another momentum indicator, Stochastic Oscillator, shows the location of the close relative to the high-low range over a set number of periods. The term "stochastic" makes use of the price range of a current price over a period of time. This method aims to forecast price turning points by comparing the closing price of a security to its price range. The KDJ indicator is an extension of Stochastic Oscillator. Compared with the Stochastic Oscillator, KDJ includes one more 'J' line along with the traditional 'D' and 'K' lines. Along with D & K, the J line assists traders in identifying overbought and oversold markets. The KDJ indicator can be used for devising trading strategies. Either a trader can buy when all lines (K, D, and J) are below 20 and sell above 80, or a trader can use the 'J' line to construct a momentum based technique. The KDJ is superior to the stochastic indicator as identifying stock overbought and oversold.

To understand the KDJ indicator, the construction of the stochastic indicator is introduced. The stochastic indicator consists of K and D lines which move between 0 and 100. The K line in the stochastic indicator is computed as follows:

$$\text{K} = \frac{close_t - Lowest_6}{Highest_6 - Lowest_6} \times 100\%$$

Where $Lowest_6$ denotes the lowest closing price of the last 6 days, $Highest_6$ denotes the highest closing price of the last 6 days.

The D line in the stochastic indicator is the simple moving average of the K line. D is based on what the trader wants to choose, most often the three day simple moving average of K. The computation of D is as follows:

$$\text{D} = \text{N day simple moving average of K line}$$

where N is usually three or a value decided by the trader.

The J line in the KDJ indicator can be computed as follows:

$$\text{J} = (3 \times \text{D}) - (2 \times \text{K})$$

In the J line, D is usually given greater weight than K. In the KDJ indicator, K is the fastest line and J is the slowest line.

Exponential smoothing is applied to smooth time series data which acts as low-pass filters to remove high frequency noise. Exponential smoothing is developed from moving average method, which does not need to store much former data,

Table 1
Taiwan weighted index in January 2000.

$Date_t$	$Index_t$	$Close_t$	ROC	Fuzzy value
2000-1-4	1	8756.55	–	–
2000-1-5	2	8849.87	1.07%	A_4
2000-1-6	3	8922.03	0.82%	A_3
2000-1-7	4	8845.47	−0.86%	A_2
2000-1-10	5	9102.6	2.91%	A_5
2000-1-11	6	8927.03	−1.93%	A_1
2000-1-12	7	9144.65	2.44%	A_5
2000-1-13	8	9107.19	−0.41%	A_2
2000-1-14	9	9023.24	−0.92%	A_2
2000-1-15	10	9191.37	1.86%	A_4
2000-1-17	11	9315.43	1.35%	A_4
2000-1-18	12	9250.19	−0.70%	A_2
2000-1-19	13	9151.44	−1.07%	A_1
2000-1-20	14	9136.95	−0.16%	A_2
2000-1-21	15	9255.94	1.30%	A_4
2000-1-24	16	9387.07	1.42%	A_4
2000-1-25	17	9372.37	−0.16%	A_2
2000-1-26	18	9581.96	2.24%	A_5
2000-1-27	19	9628.98	0.49%	A_3
2000-1-28	20	9696.91	0.71%	A_3
2000-1-29	21	9636.38	−0.62%	A_2
2000-1-31	22	9744.89	1.13%	A_4

but includes the importance of data at every period, and uses all of the historical data. Exponential smoothing method is to achieve the minimum variance (MSE) between the measured value and the predictive value, and its estimation is nonlinear.

The simplest form of exponential smoothing is given by the formula:

$$s_t = \alpha x_t + (1-\alpha)s_{t-1}$$

where α is the *smoothing factor*, and $0 < \alpha < 1$. $\{x_t\}$ denotes the raw time series data, and the output of the exponential smoothing algorithm is commonly written as $\{s_t\}$. The smoothed statistic s_t is a simple weighted average of the current observation x_t and the previous smoothed statistic s_{t-1}. The term *smoothing factor* applied to α here is something of a misnomer, as larger values of α actually reduce the level of smoothing, and in the limiting case with $\alpha = 1$ the output series is just the same as the original series. If two observations are available, it is easy to produce a smoothed statistic for simple exponential smoothing. Values of α close to one have less of a smoothing effect and give greater weight to recent changes in data, while values of α closer to zero have a greater smoothing effect and are less responsive to recent changes. There is no formally correct procedure for choosing α.

4. Fuzzy time series model

There are mainly four steps to construct a forecasting model with fuzzy time series: (1) Fuzzify the training data; (2) Extract the fuzzy logic relationships from the training data, (3) Fuzzify the testing data, and (4) Predict the testing data using the fuzzy logic relationships extracted in (2). To interpret the proposed forecasting model in detail, the Taiwan Weighted Index in January 2000 (Table 1) is taken as an example, which is similar to Chen's model [3].

Step 1: First, calculate the ROC according to the closing price of training data and define the range of the domain. Let D_{max} and D_{min} be the maximum and minimum values of the percentage change, respectively, and then the universe of discourse $U = [D_{min}, D_{max}]$, which is shown in Table 1.

Step 2: Partition the universe of discourse into isometric intervals: $u_0, u_1, u_2, \ldots, u_n$. $m_0, m_1, m_2, \ldots, m_n$ denotes the mid-values of each interval correspondingly. The partition of U is shown in next section.

Step 3: Define the fuzzy sets $A = \{A_0, A_1, A_2, \ldots, A_n\}$ as:

$$A_0 = a_{00}/u_0 + a_{01}/u_1 + \cdots + a_{0n}/u_n \tag{3}$$

$$A_1 = a_{10}/u_0 + a_{11}/u_1 + \cdots + a_{1n}/u_n \tag{4}$$

$$A_n = a_{n0}/u_0 + a_{n1}/u_1 + \cdots + a_{nn}/u_n \tag{5}$$

where $a_{i,j}$ denotes the membership degree of the interval j to the fuzzy set i, and "+" here indicates which elements are in A_i and their membership degree. The value of $a_{i,j}$ is defined as follows:

$$a_{i,j} = \begin{cases} 1 & i = j \\ 0.5 & i = j+1 \ or \ i = j-1 \\ 0 & others \end{cases}$$

Table 2
Fuzzy logic relationships group (FLRG).

Left-hand side	FLRG
First-order FLRGs	
A_1	$A_1 \to A_2$ 1 $A_1 \to A_5$ 1
A_2	$A_2 \to A_1$ 1 $A_2 \to A_2$ 1 $A_2 \to A_4$ 3 $A_2 \to A_5$ 2
A_3	$A_3 \to A_2$ 2 $A_3 \to A_3$ 1
A_4	$A_4 \to A_2$ 2 $A_4 \to A_3$ 1 $A_4 \to A_4$ 2
A_5	$A_5 \to A_1$ 1 $A_5 \to A_2$ 1 $A_5 \to A_3$ 1
Second-order FLRGs	
A_1, A_2	$(A_1, A_2) \to A_4$ 1
A_1, A_5	$(A_1, A_5) \to A_2$ 1
A_2, A_1	$(A_2, A_1) \to A_2$ 1
A_2, A_2	$(A_2, A_2) \to A_4$ 1
A_2, A_4	$(A_2, A_4) \to A_4$ 2
A_2, A_5	$(A_2, A_5) \to A_1$ 1, $(A_2, A_5) \to A_3$ 1
A_3, A_2	$(A_3, A_2) \to A_5$ 1, $(A_3, A_2) \to A_4$ 1
A_3, A_3	$(A_3, A_3) \to A_2$ 1
A_4, A_2	$(A_4, A_2) \to A_1$ 1, $(A_4, A_2) \to A_5$ 1
A_4, A_3	$(A_4, A_3) \to A_2$ 1
A_4, A_4	$(A_4, A_4) \to A_2$ 2
A_5, A_1	$(A_5, A_1) \to A_5$ 1
A_5, A_2	$(A_5, A_2) \to A_2$ 1
A_5, A_3	$(A_5, A_3) \to A_3$ 1
Third-order FLRGs	
A_4, A_3, A_2	$(A_4, A_3, A_2) \to A_5$ 1
A_3, A_2, A_5	$(A_3, A_2, A_5) \to A_1$ 1
A_2, A_5, A_1	$(A_2, A_5, A_1) \to A_5$ 1
A_5, A_1, A_5	$(A_5, A_1, A_5) \to A_2$ 1
A_1, A_5, A_2	$(A_1, A_5, A_2) \to A_2$ 2
A_5, A_2, A_2	$(A_5, A_2, A_2) \to A_4$ 1
A_2, A_2, A_4	$(A_2, A_2, A_4) \to A_4$ 1
A_2, A_4, A_4	$(A_2, A_4, A_4) \to A_2$ 2
A_4, A_4, A_2	$(A_4, A_4, A_2) \to A_1$ 1, $(A_4, A_4, A_2) \to A_5$ 1
A_4, A_2, A_1	$(A_4, A_2, A_1) \to A_2$ 1
A_2, A_1, A_2	$(A_2, A_1, A_2) \to A_4$ 1
A_1, A_2, A_4	$(A_1, A_2, A_4) \to A_4$ 1
A_4, A_2, A_5	$(A_4, A_2, A_5) \to A_3$ 1
A_2, A_5, A_3	$(A_2, A_5, A_3) \to A_3$ 1
A_5, A_3, A_3	$(A_5, A_3, A_3) \to A_2$ 1
A_3, A_3, A_2	$(A_3, A_3, A_2) \to A_4$ 1

Step 4: Calculate the membership degree of the daily percentage change for each element in A, and fuzzify it to A_i whose value is the biggest to form the fuzzy sequence shown in the fifth column of Table 1.

Step 5: Extract the first-order, second-order and third-order fuzzy logic relationships (FLR) from the fuzzy sequence, and classify them by the left-hand sides to form corresponding fuzzy logic relationships group (FLRG). Each FLR is followed by its frequency, as shown in Table 2. It is noted that Table 2 shows all of the first-order, second-order, and third order fuzzy logic relationships in Table 1. In fact, some FLRGs are not used for forecasting. For example, when we forecast the value of 2000-2-1, we use only first-order FLRGs: $A_4 \to A_2$, $A_4 \to A_3$, $A_4 \to A_4$, and second-order FLRGs: $(A_2, A_4) \to A_4$, but no third-order FLRGs (A_3, A_2, A_4) occurs. If the target is to extract a k-th order FLRs, the left-hand side of FLRGs is fuzzy sets of k days, and the right hand side is the corresponding fuzzy set.

Step 6: Build the weighted matrix W, and normalize W by row to obtain W'. Take the first-order fuzzy logic relationships group for example (see Fig. 1). The left-hand side is shown in the ordinate axis of the weighted matrix and the right-hand is shown in the abscissa axis. Each number in W is the frequency of occurrence of the corresponding FLR. W' is the normalized W.

Then, the weight value of the fuzzy logic relationship $A_i \to A_j$ is $W'_{i,j}$. The processing methods for the second-order and third-order fuzzy logic relationships group are similar.

Step 7: Fuzzify the testing data. The method is similar as those above and predicts the stock's closing price in the future based on the normalized weighted matrix W'. First, for the first-order fuzzy logic relationships group, a specific prediction method is given as follows:

$$W = \begin{array}{c} \\ A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{array}{c} \begin{array}{cccccc} A_0 & A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 3 & 2 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \quad W' = \begin{array}{c} \\ A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{array}{c} \begin{array}{cccccc} A_0 & A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 1/7 & 1/7 & 0 & 3/7 & 2/7 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 2/5 & 1/5 & 2/5 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \end{bmatrix} \end{array}$$

Fig. 1. Weighted matrixes W and W'.

If the fuzzy value of day t is A_i, and the FLRG whose left-hand side is A_i contains$\{A_i \to A_{k_1},\ A_i \to A_{k_2},\ \ldots,\ A_i \to A_{k_p}\}$, then the predicted fuzzy value of day t+1 is $W'_{i,k_1} \times m_{k_1} + W'_{i,k_2} \times m_{k_2} + \cdots + W'_{i,k_p} \times m_{k_p} = \sum_{j=1}^{p} W'_{i,k_j} \times m_{k_j}$, where m_{k_j} is the mid-value of interval A_{k_j}. If there is no FLRG whose left-hand side is A_i, then the predicted fuzzy value of day $t+1$ is m_i.

For the second-order, we use the fuzzy values of the previous two days for prediction, and specific method is given as follows:

If the fuzzy value of day t-1 and day t is $(A_i,\ A_j)$, and the FLRG whose left-hand side is $(A_i,\ A_j)$ contains $\{(A_i,\ A_j) \to A_{k_1},\ (A_i,\ A_j) \to A_{k_2},\ \ldots,\ (A_i,\ A_j) \to A_{k_p}\}$, then the predicted fuzzy value of day t+1 is $W'_{i,k_1} \times m_{k_1} + W'_{i,k_2} \times m_{k_2} + \cdots + W'_{i,k_p} \times m_{k_p} = \sum_{j=1}^{p} W'_{i,k_j} \times m_{k_j}$, where m_{k_j} is the mid-value of interval A_{k_j}. If there is no FLRG whose left-hand side is $(A_i,\ A_j)$, then we do not use the second-order fuzzy logic relationships group, just use the first-order to predict. In general, the amount of data is very large, such that this type of situation may not happen.

For the third-order, we apply the fuzzy values of the previous three days to predict. Forecasting method is similar as above.

Finally, we average the three predicted fuzzy values to achieve the final predicted value.

5. A new forecasting method based on technical analysis and genetic algorithm

In this section, a new method for financial forecasting is presented by combining multi-order fuzzy time series, technical analysis, and genetic algorithm. The model uses genetic algorithm operators such as selection, crossover, and mutation to iteratively seek for an optimal domain partition. There is one only chromosome for each individual and each chromosome stores genetic information that reflects a type of domain partition. Every gene in the chromosome corresponds to an interval of the partition. The model searches for a good domain partition using the training data. Using the fuzzy time series, the model can obtain different predicted values with each partition. The model takes the root mean square error between the predicted value and the actual value as the fitness of the corresponding individual.

We also use the Taiwan Weighted Index from January 2000 (Table 1) as an example to elaborate. The proposed method is now presented as follows:

Step 1: Construct the fuzzy time series model.

(1) Encoding. $Group_k = \{C_j | j = 1, 2, \ldots, m; k = 1, 2, \ldots, T\}$ is the whole population, where C_j is the jth chromosome in the population, m is the size of the population, and k is the current number of iterations. Each C_j consists of four genes: $g(ROC)$,$g(MACD)$,$g(K)$ and $g(J)$. Taking $g(ROC)$ as an example, $g(ROC)_{j,i} = (v_{j,i}, v_{j,i+1}]$ is an encoding gene, where $v_{j,i}$ is a break point. And two break points establish a fragment as genetic information.

Domain partition is the key of the population encoding. If there are n+1 gene segments, the universe of discourse U will be divided into $n+1$ intervals. $\{v_i | i = 1, 2, \ldots, n\}$ denotes the value of the break points, then $U = (-\infty, v_1] \cup (v_1, v_2] \cup \ldots \cup (v_{n-1}, v_n] \cup (v_n, +\infty]$. Therefore, a chromosome has a corresponding domain partition, and the population presents a set of domain partitions. The description of population encoding in the genetic algorithm is shown in Fig. 2. The corresponding partition of the universe of discourse in Table 1 is shown in Fig. 3.

(2) Fitness calculation. Based on the FLRGs and the normalized weighted matrix W', the model can obtain the predicted price. The fitness of each individual is RMSE between the actual value ($Real_t$) and the predicted value ($Predict_t$) in the tth day. The formula is listed below:

$$RMSE = \sqrt{\sum_{t=1}^{n} (Predict_t - Real_t)^2 / n} \tag{6}$$

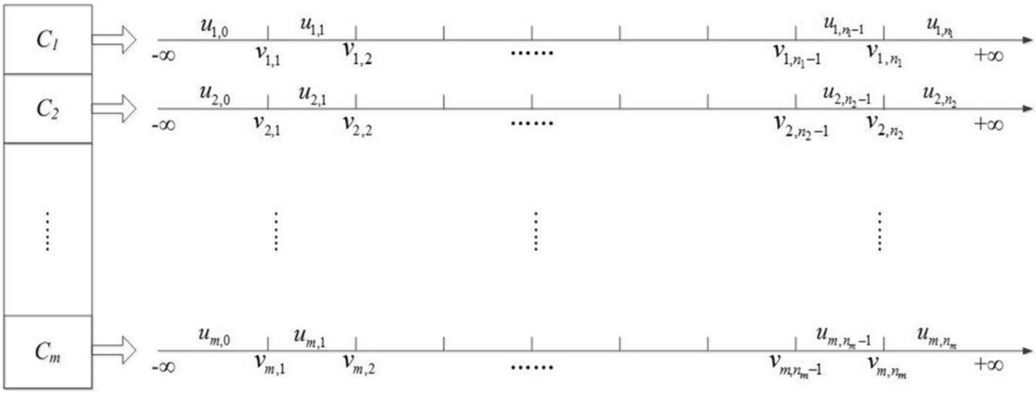

Fig. 2. The description of population encoding.

u_0 u_1 u_2 u_3 u_4 u_5

$(-\infty, -2\%]$	$(-2\%, -1\%]$	$(-1\%, 0\%]$	$(0\%, 1\%]$	$(1\%, 2\%]$	$(2\%, +\infty]$

Fig. 3. The domain partition of *U*.

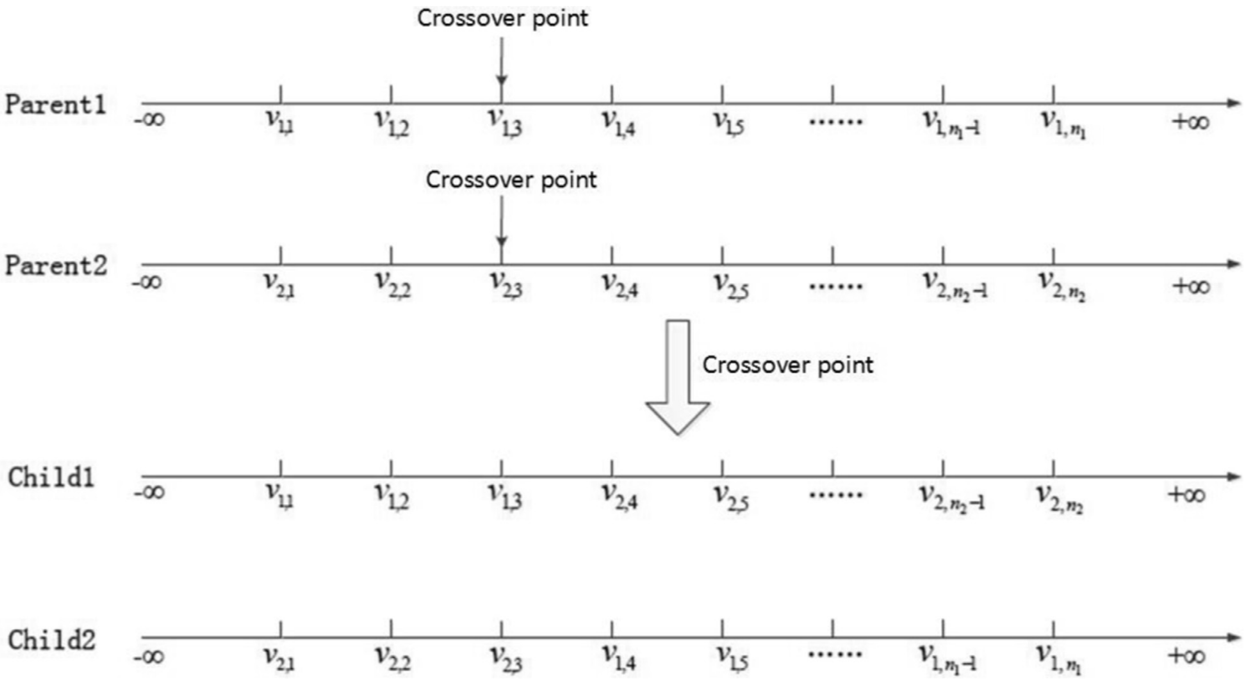

Fig. 4. The crossover process.

Step 2: The selection process. The model uses the tournament method to select superior individuals. The method randomly selects m individuals from the population and chooses the two optimal individuals, whose fitness is better than others, as the parents to process the crossover.

Step 3: The crossover process. The model uses single-point crossover operator to hybridize two parent individuals. For two parent individuals *Parent1* and *Parent2*, we randomly select a break point from each of them as the crossover point; the crossover points are represented as *p1* and *p2*, respectively. Then, the first half of *Parent1* before *p1* and the second half of *Parent2* after *p2* are merged into a new individual *Child1* and the first half of *Parent2* before *p2* and the second half of *Parent1* after *p1* are merged into another new individual *Child2*. To form two new domain partitions, break points on *Child1* and *Child2* need to be sorted in ascending order. The process is illustrated in Fig. 4.

Step 4: The mutation process. The mutation process is implemented with the probability P_M for genic change. There are three strategies for mutation as shown in Fig. 5:

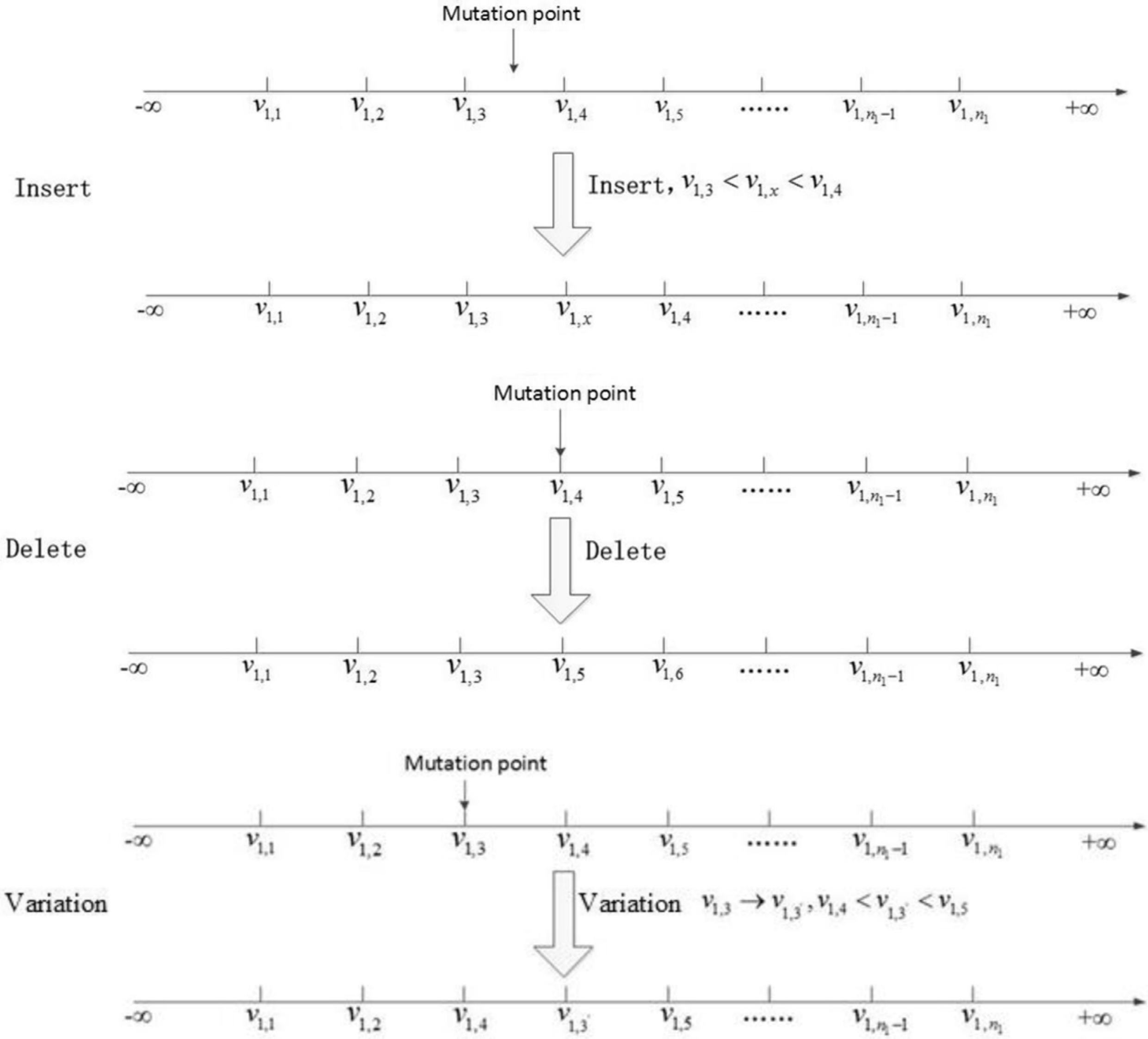

Fig. 5. The three ways of the mutation process.

(1) Insertion. Randomly generate one break point v_k in $[D_{min}, D_{max}]$ and insert it into the chromosome properly to make the points in the chromosome maintain ascending order.
(2) Deletion. Randomly delete a break point v_k from the chromosome if the number of break points in the chromosome is greater than two.
(3) Variation. Randomly select a break point v_k and change its value. Then, adjust the position of v_k to make the points in the chromosome maintain ascending order.

Step 5: Check whether the termination condition is satisfied. In the experiment, we use two rules as stop conditions:

(1) The number of iterations is beyond the maximum iteration times T;
(2) The best fitness remains unchanged.

The iterative process of the genetic algorithm is stopped if any of the above two rules are met and the algorithm goes to step 6; Otherwise, the iterative process continues.

Step 6: Let C_{best} be the best individual evolved from the iterative process of the genetic algorithm and partition the universe of discourse based on the C_{best}. Then, fuzzify the training data according to C_{best}, extract the fuzzy logic relationships group *FLRGs* and establish the normalized weighted matrix W'.

Step 7: Fuzzify the testing data according to C_{best}, then predict the value according to the *FLRGs* and W' of step 6. Finally calculate the RMSE between the actual value and the predicted value.

Table 3
FLRG and frequency of occurrence.

FLRG	Frequency	FLRG	Frequency	FLRG	Frequency
$(A_1, M_1) \to A_1$	2	$(A_1, A_2, K_1) \to A_1$	4	$(A_1, J_1) \to A_1$	2
$(A_1, A_3, M_2) \to A_2$	2	$(A_1, K_2) \to A_2$	4	$(A_1, A_1, J_2) \to A_2$	2
$(A_1, M_1) \to A_2$	3	$(A_1, K_1) \to A_1$	5	$(A_2, J_1) \to A_1$	3
$(A_1, A_2, M_1) \to A_2$	3	$(A_2, K_2) \to A_2$	5	$(A_1, J_2) \to A_2$	3
$(A_1, M_2) \to A_1$	3				

For example, assume that *FLRGs* is as Table 2. Flags and frequency of occurrence are shown in Table 3. For day *t*, the fuzzy value of ROC, MACD, K, and J is $A_2, M_1, K_2,$ and J_1, respectively. FLRGs are as follows: $(A_1, A_2, M_1) \to A_2$ 3, $(A_2, K_2) \to A_2$ 5, $(A_2, J_1) \to A_1$ 3, then the predicted fuzzy value of day $t+1$ is $(3 \times m_1 + 8 \times m_2)$ / $(3+8)$, where m_1, m_2 is the mid-value of interval A_1, A_2. Based on the predicted value, we can compute the fitness value and RMSE because ROC is the main variable, and MACD and KDJ are the secondary variables. In the process of extracting FLRG, MACD and KDJ are combined with ROC to construct the left side of FLR, and the right side is the corresponding fuzzy set of ROC. Instead of orders of k fuzzy sets in Table 2, the left sides of FLRs in Table 3 summarize all fuzzy sets in k continuous days. $(\mathrm{A}_1, \mathrm{A}_3, \mathrm{M}_2) \to \mathrm{A}_2$ denotes that in continuous k days, there exist $\mathrm{A}_1, \mathrm{A}_3, \mathrm{M}_2$, and in the $\mathrm{k}+1$ day, the fuzzy set of ROC is A_2.

6. Experiment results

In this section, our proposed method is used to forecast six well-known financial time series: TAIEX, Dow Jones, NASDAQ, HSI, SP500, and the NTD/USD exchange rates. We implemented the proposed method using the C++ programming language on an Intel Core i5 PC. In our model, we determine the parameters with a great deal of experiments based on simulation and its selection is based on how much accuracy could provide among other time series in the competition. The selected parameters used in genetic algorithm are set as follows: the maximum number of iterations T is set as 100; the size of the population m is set as 200; the crossover probability P_C is set as 80%; the mutation probability P_M is set as 1%; and the individual number in the tournament method K is set as 6. Because of the randomness in genetic algorithm, we executed the proposed method 100 times and used the average values as the results.

6.1. Index forecasting

To compare the proposed method with most well-known models in this field, the daily closing prices from 1990 to 2004 are used as the evaluation dataset. The data from January to October are used as the training set, and the data from November to December are used as the testing set to verify the performance of the model. The compared models include not only fuzzy time series models but also neural network models [3,26,29–31,40,46,47,52,53]. RMSE is used to evaluate the performance of the proposed method. The RMSE is smaller and the results are better. We also give the directional accuracy rate (DAR) of the forecast results for each year. Here, the directional accuracy rate is defined as the ratio in the same direction of the predicted value and actual value of the next moment relative to the actual value of the previous moment; the formula is as follows:

$$DAR = \sum_{i=1}^{n} d_i / n \times 100\% \tag{7}$$

$$d_i = \begin{cases} 1, & if\ (Predict_i - Real_{i-1}) \times (Real_i - Real_{i-1}) \ \geq 0 \\ 0, & otherwise \end{cases} \tag{8}$$

Table 4 shows the comparison of the RMSEs and the average RMSEs of different methods on forecasting the TAIEX from 1999 to 2004. Table 5 shows the comparison of the RMSEs and the average RMSEs of different methods on forecasting the TAIEX from 1990 to 1999.

To highlight the effectiveness of the proposed method, we averaged the average RMSEs of the two sections (1999–2004 and 1990–1999) for the same model, as shown in Table 6.

From Tables 4 and 5 we can observe that, although our predicted average RMSE from 1999 to 2004 is a bit higher than the result in [53], all of the other average RMSEs are smaller than those of the existing works in [3,26,29–31,40,46,47,52,53]. Because of the multivariate strategy, our predicted results tend to be conservative. At the time of improving the directional accuracy rate, it also increases the deviation of each single value. This is the direct cause of a bit higher RMSE. However, from Table 6, our average RMSE of the two sections outperforms all of the results of other methods. Overall, our method is more effective. In addition, we can observe that the improvement of the results becomes more and more difficult beginning from the fourth model (Chen et al.'s method [31]).

Furthermore, four well-known financial time series, the Dow Jones, NASDAQ, HSI, and SP500, are applied to verify the effectiveness of our method and compared the generalization ability of different models. For comparison, we did the same

Table 4
A comparison of the RMSEs of different methods on forecasting TAIEX from 1999 to 2004.

		1999	2000	2001	2002	2003	2004	Average RMSEs
Huarng et al.'s method [26] (Use NASDAQ)		N/A	158.70	136.49	95.15	65.51	73.57	105.88
Huarng et al.'s method [26] (Use Dow Jones)		N/A	165.80	138.25	93.73	72.95	73.49	108.84
Huarng et al.'s method [26] (Use M1b)		N/A	169.19	133.26	97.10	75.23	82.01	111.36
Huarng et al.'s method [26] (Use NASDAQ & Dow Jones)		N/A	157.64	131.98	93.48	65.51	73.49	104.42
Huarng et al.'s method [26] (Use NASDAQ & M1b)		N/A	155.51	128.44	97.15	70.76	73.48	105.07
Huarng et al.'s method [26] (Use NASDAQ & Dow Jones & M1b)		N/A	154.42	124.02	95.73	70.76	72.35	103.46
Chen's fuzzy time series model (U_FTS Model) [3,30,31]		120.00	176.00	148.00	101.00	74.00	84.00	117.40
Univariate conventional regression model (U_R model) [30,31]		164.00	420.00	1070.00	116.00	329.00	146.00	374.20
Univariate neural network model (U_NN model) [30,31]		107.00	309.00	259.00	78.00	57.00	60.00	145.00
Univariate neural network-based fuzzy time series model [30,31,46]		109.00	255.00	130.00	84.00	56.00	116.00	125.00
Univariate neural network-based fuzzy time series model use substitutes (U_NN_FTS_S model) [30,31,46]		109.00	152.00	130.00	84.00	56.00	116.00	107.80
Bivariate conventional regression model (B_R model) [30,31]		103.00	154.00	120.00	77.00	54.00	85.00	98.80
Bivariate neural network model (B_NN model) [30,31]		112.00	274.00	131.00	69.00	52.00	61.00	116.40
Bivariate neural network-based fuzzy time series model [30,31]		108.00	259.00	133.00	85.00	58.00	67.00	118.30
Bivariate neural network-based fuzzy time series model use substitutes (B_NN_FTS_S model) [30,31]		112.00	131.00	130.00	80.00	58.00	67.00	96.40
AR (1) model [40]		116.84	155.12	112.39	97.09	91.67	79.94	108.84
AR (2) model [40]		128.15	142.30	129.84	89.80	66.58	60.33	102.83
Chen and Chang's method [29]	Use NASDAQ	123.64	131.10	115.08	73.06	66.36	60.48	94.95
	Use Dow Jones	101.97	148.85	113.70	79.81	64.08	82.32	98.46
	Use M1B	156.92	142.70	132.76	96.06	90.27	100.10	119.80
	Use Dow Jones & NASDAQ	106.34	130.13	113.33	72.33	60.29	68.07	91.75
	Use NASDAQ & M1B	116.22	134.63	116.59	76.48	53.51	69.29	94.45
	Use NASDAQ & Dow Jones & M1B	111.70	129.42	113.67	66.82	56.10	64.76	90.41
Chen and Chen's method [30]	Use Dow Jones	115.47	127.51	121.98	74.65	66.02	58.89	94.09
	Use NASDAQ	119.32	129.87	123.12	71.01	65.14	61.94	95.07
	Use M1b	120.01	129.87	117.61	85.85	63.10	67.29	97.29
	Use Dow Jones & NASDAQ	116.64	123.62	123.85	71.98	58.06	57.73	91.98
	Use Dow Jones & M1B	116.59	127.71	115.33	77.96	60.32	65.86	93.96
	Use NASDAQ & M1B	114.87	128.37	123.15	74.05	67.83	65.09	95.56
	Use NASDAQ & Dow Jones & M1b	112.47	131.04	117.86	77.38	60.65	65.09	94.08
Chen et al.'s method [31]	Use Dow Jones	102.34	131.25	113.62	65.77	52.23	56.16	86.89
	Use NASDAQ	102.11	131.30	113.83	66.45	52.83	54.17	86.78
	Use M1b	103.52	131.36	112.55	66.23	53.20	55.36	87.04
Chen and Kao's method [46]		87.63	125.34	114.57	76.86	54.29	58.17	86.14
FTSGA model [47]		102.74	126.68	115.79	65.56	57.40	56.10	87.38
ACO-AR model [53]		102.22	131.53	112.59	60.33	51.54	50.33	**84.75**
Chen et al.'s method [52]	Use Dow Jones	103.90	127.32	115.37	64.71	52.84	53.36	86.25
	Use NASDAQ	104.99	124.52	114.66	64.79	53.63	52.96	85.93
	Use M1b	105.61	127.37	115.46	66.07	53.67	53.3	86.91
The proposed method		101.29	125.42	113.22	63.99	52.99	52.40	84.88

forecast using the models FTSGA in [47] and ACO-AR in [53] respectively. Section 1 introduces the difference of three models. The experimental results are shown in Tables 7–11.

Table 7 shows the comparison of the RMSEs and the average RMSEs for forecasting the five indexes from 1999 to 2004. Table 8 shows the comparison of the RMSEs and the average RMSEs for forecasting the five indexes from 1990 to 1999. Table 9 shows the comparison of the DARs for forecasting the five indexes from 1999 to 2004. Table 10 shows the comparison of the DARs for forecasting the five indexes from 1990 to 1999.

According to Tables 7 and 8, comparing FTSGA and ACO-AR, our proposed method obtains four better average RMSEs in five indexes, and performs not well for TAIEX in Table 7 and HSI in Table 8. From Tables 9 and 10, we can observe that, comparing FTSGA and ACO-AR, our proposed method obtains four better average DARs in five indexes for stock indexes from 1999 to 2004. Our proposed method obtains four better average DARs in five indexes for stock indexes from 1990 to 1999. On the whole, our method is more effective and stable.

Similarly, we also reported the average RMSEs and DARs of the two sections (1999–2004 and 1990–1999) of the five indexes, as shown in Table 11. From Table 11, comparing FTSGA and ACO-AR, the proposed method obtains five better average

Table 5
A comparison of the RMSEs of different methods on forecasting TAIEX from 1990 to 1999.

		1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	Average RMSEs
Conventional models [45]	Average-based lengths	220.00	80.00	60.00	110.00	112.00	79.00	54.00	148.00	167.00	149.00	117.90
	Distribution-based lengths	270.00	79.00	60.00	105.00	132.00	79.00	52.00	149.00	159.00	159.00	124.40
Weighted models [45]	Average-based lengths	227.00	61.00	67.00	105.00	135.00	70.00	54.00	133.00	151.00	142.00	114.50
	Distribution-based lengths	266.00	67.00	56.00	105.00	114.00	70.00	52.00	152.00	154.00	145.00	118.10
Chen and Chen's method [30]	Use Dow Jones	172.89	72.87	43.44	103.21	78.63	66.66	59.75	139.68	124.44	115.47	97.70
	Use NASDAQ	169.93	66.12	49.61	104.75	75.66	67.01	60.90	140.86	144.13	119.32	99.83
	Use Dow Jones & NASDAQ	172.99	74.85	43.78	101.38	78.13	68.14	61.26	139.29	132.94	116.64	98.94
AR (1) model [40]		178.08	77.23	85.34	101.23	90.85	82.38	78.63	146.22	144.53	116.84	110.13
AR (2) model [40]		198.24	65.38	85.30	113.40	97.16	73.63	57.85	174.09	135.21	128.15	112.84
Chen et al.'s method [49]	Use Dow Jones	174.35	43.78	43.12	108.02	88.32	53.69	51.02	139.86	113.58	102.34	91.81
	Use NASDAQ	176.17	43.16	43.34	106.66	87.95	53.30	51.10	138.41	113.88	102.11	91.61
Chen and Kao's method [46]		156.47	56.50	36.45	126.45	105.52	62.57	51.50	125.33	104.12	87.63	91.25
FTSGA model [47]		175.80	44.27	42.62	102.33	78.45	56.36	48.80	134.68	112.96	102.86	89.91
ACO-AR model [53]		187.10	39.58	39.37	101.80	76.32	56.05	49.45	123.98	118.41	102.34	89.44
Chen et al.'s method [52]	Use Dow Jones	180.36	43.8	43.06	104.89	75.35	55.06	50.06	133.82	112.11	103.9	90.24
	Use NASDAQ	174.15	45.04	42.10	104.94	76.40	54.96	50.17	133.45	113.37	104.99	89.96
The proposed method		189.30	41.74	38.46	103.72	61.90	48.85	50.72	115.77	114.21	110.09	**87.47**

Table 6
A comparison of the average RMSEs of different methods in Tables 4 and 5.

Methods	Average RMSEs
AR (1) model [40]	109.49
AR (2) model [40]	107.84
Chen and Chen's method [30]	96.70
Chen et al.'s method [31]	89.31
Chen et al.'s method [49]	88.83
Chen and Kao's method [46]	88.70
FTSGA model [47]	88.65
Chen et al.'s method [52] using Dow Jones	88.25
Chen et al.'s method [52] using NASDAQ	87.95
ACO-AR model [53]	87.10
The proposed method	**86.18**

Table 7
A comparison of the RMSEs of three methods for forecasting five indexes from 1999 to 2004.

	Methods	1999	2000	2001	2002	2003	2004	Average RMSEs
TAIEX	The proposed method	101.29	125.42	113.22	63.99	52.99	52.40	84.88
	FTSGA model [47]	102.74	126.68	115.79	65.56	57.40	56.10	87.38
	ACO-AR model [53]	102.22	131.53	112.59	60.33	51.54	50.33	**84.75**
Dow Jones	The proposed method	67.67	105.46	91.53	85.47	48.73	63.28	**77.02**
	FTSGA model [47]	82.20	130.50	96.72	106.55	57.79	64.10	89.64
	ACO-AR model [53]	84.29	130.21	90.01	102.70	57.22	55.97	86.73
NASDAQ	The proposed method	45.43	105.22	32.46	23.71	19.39	14.02	**40.04**
	FTSGA model [47]	49.14	104.49	35.77	26.63	22.05	16.36	42.41
	ACO-AR model [53]	46.04	121.72	31.74	25.41	21.99	15.48	43.73
HSI	The proposed method	220.87	240.31	159.27	95.01	127.21	81.98	**154.11**
	FTSGA model [47]	216.87	246.12	163.33	104.28	129.00	101.29	160.15
	ACO-AR model [53]	225.24	253.21	168.45	94.97	116.42	102.65	160.16
SP500	The proposed method	9.50	19.19	10.34	9.24	5.58	7.11	**10.16**
	FTSGA model [47]	11.38	19.60	11.39	11.71	6.49	7.14	11.28
	ACO-AR model [53]	10.14	19.17	9.945	11.43	6.364	6.413	10.58

RMSEs and DARs in five indexes. The results of RMSEs and DARs in Table 11 are visualized in Figs. 6 and 7, respectively. From Figs. 6 and 7, we can clearly observe that the proposed model outperforms FTSGA and ACO-AR for the five indexes.

From the above comparisons, we can observe that, for the average RMSEs and the average DARs of five indices in different period, there exists an index that the proposed method performs not well. Although our experimental results are not always the best, most of them outperform the results of the latest models in [47] and [53]. Moreover, this type of phenomenon is very common in this field of stock prediction because a particular prediction method is not necessarily suitable for all types of stocks. Therefore, these additional experiments we performed can also offer evidence for the efficiency of our proposed method. In particular, for the average RMSEs and Dars of different period, the proposed method outperforms

Table 8
A comparison of the RMSEs of three methods for forecasting five indexes from 1990 to 1999.

	Methods	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	Average RMSEs
TAIEX	The proposed method	189.30	41.74	38.46	103.72	61.90	48.85	50.72	115.77	114.21	110.09	**87.47**
	FTSGA model [47]	175.80	44.27	42.62	102.33	78.45	56.36	48.80	134.68	112.96	102.86	89.91
	ACO-AR model [53]	187.10	39.58	39.37	101.80	76.32	56.05	49.45	123.98	118.41	102.34	89.44
Dow Jones	The proposed method	20.43	30.32	15.99	16.81	26.99	30.98	47.83	79.14	91.66	84.26	**44.44**
	FTSGA model [47]	21.19	29.41	16.30	18.09	27.53	30.93	46.89	82.60	94.51	82.09	44.95
	ACO-AR model [53]	21.59	31.84	16.53	15.60	28.09	32.43	46.73	84.01	96.46	87.68	46.10
NASDAQ	The proposed method	3.23	6.93	4.04	7.07	6.72	10.54	9.76	22.86	21.07	45.41	**13.76**
	FTSGA model [47]	3.33	6.08	3.89	5.71	5.28	10.87	11.21	20.63	31.23	49.27	14.75
	ACO-AR model [53]	3.06	5.86	3.73	4.31	5.24	11.58	9.64	18.44	29.68	46.65	13.82
HSI	The proposed method	24.96	36.27	126.58	201.57	129.83	72.98	139.49	251.36	194.28	225.41	140.27
	FTSGA model [47]	26.60	38.10	126.99	193.30	133.81	70.52	145.82	248.69	202.66	217.09	140.36
	ACO-AR model [53]	24.77	36.49	138.13	192.48	128.84	69.74	136.02	260.03	196.92	217.66	**140.11**
SP500	The proposed method	2.78	4.094	2.24	2.22	2.69	2.73	6.80	10.44	11.44	9.50	**5.49**
	FTSGA model [47]	2.70	3.93	1.95	2.02	2.74	3.67	5.64	9.64	13.04	11.38	5.67
	ACO-AR model [53]	2.82	4.09	2.03	1.76	2.70	3.48	5.70	10.12	13.56	10.87	5.71

Table 9
A comparison of the DARs of three methods for forecasting five indexes from 1999 to 2004.

	Methods	1999	2000	2001	2002	2003	2004	Average DARs
TAIEX	The proposed method	65.12	75.56	51.22	63.41	48.78	53.49	59.60
	FTSGA model [47]	65.24	76.86	52.06	62.35	48.07	55.43	60.00
	ACO-AR model [53]	72.22	81.08	52.63	63.16	50.00	52.50	**61.93**
Dow Jones	The proposed method	65.85	69.21	55.36	69.23	61.54	58.54	**63.29**
	FTSGA model [47]	56.47	57.79	57.87	64.41	56.68	60.37	58.93
	ACO-AR model [53]	55.26	66.67	52.78	61.11	58.33	60.53	59.11
NASDAQ	The proposed method	78.05	69.23	58.95	61.54	51.28	66.95	**64.33**
	FTSGA model [47]	63.15	68.76	56.76	58.83	49.13	51.16	57.97
	ACO-AR model [53]	63.16	66.67	58.33	58.33	55.56	52.63	59.11
HSI	The proposed method	70.73	58.97	64.10	66.67	61.54	51.79	**62.30**
	FTSGA model [47]	69.00	58.98	61.18	60.30	62.99	44.04	59.42
	ACO-AR model [53]	71.05	55.56	58.33	63.89	66.67	48.72	60.70
SP500	The proposed method	63.51	58.97	53.85	64.10	61.54	53.00	**59.16**
	FTSGA model [47]	59.96	60.20	51.04	58.17	52.23	51.86	55.58
	ACO-AR model [53]	55.26	66.67	55.56	58.33	58.33	50.00	57.36

Table 10
A comparison of the DARs of three methods for forecasting five indexes from 1990 to 1999.

	Methods	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	Average DARs
TAIEX	The proposed method	55.56	55.44	70.45	54.34	63.00	53.19	47.92	54.28	60.47	74.42	**58.91**
	FTSGA model [47]	55.31	57.86	60.92	58.12	53.77	58.31	44.86	50.08	59.10	65.31	56.37
	ACO-AR model [53]	57.14	54.76	63.41	55.81	51.11	50.24	55.56	46.51	54.05	72.22	56.08
Dow Jones	The proposed method	47.36	66.67	55.10	51.22	52.50	61.54	69.95	64.10	69.85	53.66	59.19
	FTSGA model [47]	47.69	66.78	55.87	54.00	52.81	61.78	69.43	62.21	67.78	56.54	59.49
	ACO-AR model [53]	47.22	66.67	64.86	52.63	56.76	55.56	69.44	66.67	62.16	55.26	**59.72**
NASDAQ	The proposed method	60.10	71.79	72.55	73.17	62.50	58.97	64.10	66.67	75.95	78.05	**68.39**
	FTSGA model [47]	61.91	68.24	76.43	83.83	67.48	55.61	66.08	64.46	49.74	63.14	65.69
	ACO-AR model [53]	61.11	69.44	75.68	86.84	70.27	50.00	58.33	63.89	48.65	63.16	64.74
HSI	The proposed method	57.41	56.41	68.29	69.05	77.50	76.32	58.97	56.41	43.90	68.29	**63.26**
	FTSGA model [47]	63.59	53.72	66.89	70.71	72.79	75.86	57.49	56.35	44.92	68.97	63.13
	ACO-AR model [53]	50.00	55.56	63.16	69.23	78.38	71.43	58.33	52.78	42.11	71.05	61.20
SP500	The proposed method	43.59	58.67	64.93	60.98	62.93	58.97	58.97	61.54	65.00	63.49	**59.91**
	FTSGA model [47]	43.65	55.39	60.26	67.76	68.86	54.68	66.14	57.03	59.33	59.95	59.30
	ACO-AR model [53]	41.67	58.33	64.86	65.79	67.57	50.00	75.00	55.56	51.35	55.26	58.54

other existing models. Additionally, the average Dars of different stock-indexes are all around or higher than 60% which is a good forecasting result in this research.

6.2. NTD/USD exchange rate forecasting

Exchange rate forecasting is a challenging task in financial forecasting; therefore, our method was applied to the NTD/USD exchange rate forecasting to show performance of the proposed method. To compare the experimental results of our proposed method with the methods presented in [48] and [49], the historical data of the NTD/USD exchange rates from March

Table 11
A comparison of the average RMSEs and DARs of three methods in Tables 9 and 10.

Indexes	Methods	Average RMSEs	Average DARs
TAIEX	The proposed method	**86.18**	**59.26**
	FTSGA model [47]	88.65	58.19
	ACO-AR model [53]	87.10	59.01
Dow Jones	The proposed method	**60.73**	**61.24**
	FTSGA model [47]	67.30	59.21
	ACO-AR model [53]	66.42	59.42
NASDAQ	The proposed method	**26.90**	**66.36**
	FTSGA model [47]	28.58	61.83
	ACO-AR model [53]	28.78	61.93
HSI	The proposed method	**147.19**	**62.78**
	FTSGA model [47]	150.25	61.28
	ACO-AR model [53]	150.14	60.95
SP500	The proposed method	**7.83**	**59.54**
	FTSGA model [47]	8.48	57.44
	ACO-AR model [53]	8.15	57.95

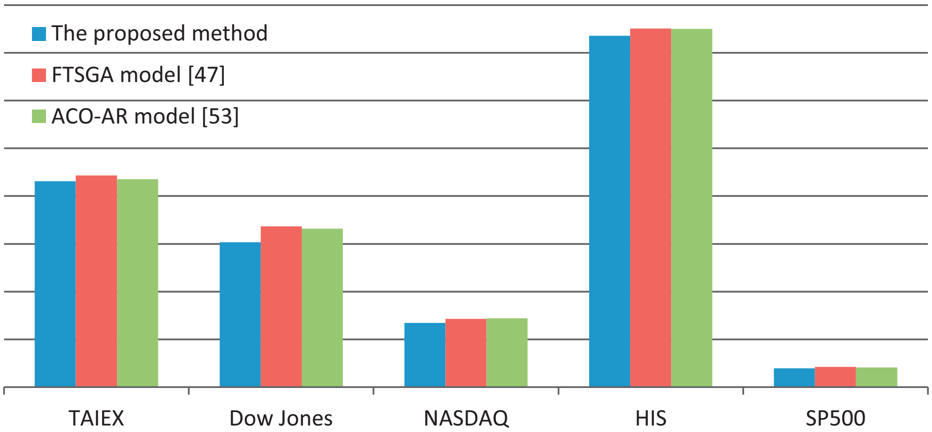

Fig. 6. A comparison of the average RMSEs of the three methods on forecasting five indexes.

1, 2006 to March 1, 2007 are used as the verification dataset and the data form March 1, 2006 to October 26, 2006 are used as the training data set. The data from October 27, 2006, to March 1, 2007 are used as the testing data set to verify the performance of the different models. Moreover, we also applied the proposed method for three-day forecasting, five-day forecasting, and seven-day forecasting which is proposed in [48]. The mean squared error (MSE) is used to evaluate the performance of the proposed method, which is defined as follows:

$$MSE = \sum_{t=1}^{n} (Predict_t - Real_t)^2 / n \tag{9}$$

where n denotes the number of dates needed to be forecasted.

Table 12 shows the comparison of the average MSEs for different methods on forecasting the NTD/USD exchange rates from March 1, 2006 to March 1, 2007. Similar to the experiments in index forecasting, we averaged the four predicted values of different methods, as shown in Table 13.

From Table 12, we can observe that for one-day, three-day, five-day and seven-day forecasting, our proposed method achieves the best result and outperforms all the other models. This is enough to demonstrate the efficiency of our proposed method. Moreover, Table 13 shows that our proposed method has the smallest average MSE among all of the methods. That is, the proposed method has obvious advantages compared with the methods presented in [48] and [49].

7. Conclusions

In this paper, a novel financial forecasting model is presented based on multi-order fuzzy time series, technical analysis, and a genetic algorithm. The proposed model exploits a genetic algorithm to iteratively seek for the optimal partition method of the universe of discourse. Compared with traditional fuzzy time series models of this field, in addition to the first-order fuzzy time series, we use a hybrid multi-order fuzzy time series to forecast financial time series. Technical analysis is used to construct multivariate fuzzy time series to show the effect of different technical indicators. Exponential

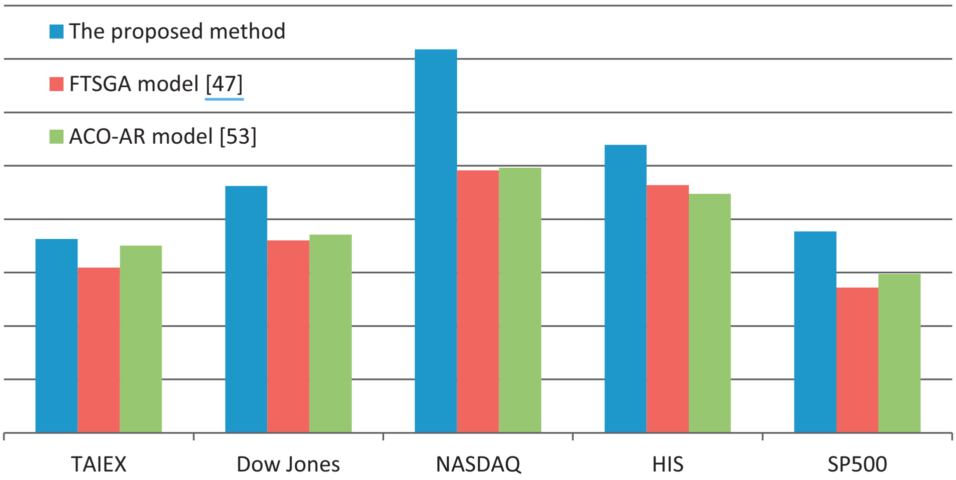

Fig. 7. A comparison of the average DARs of the three methods on forecasting five indexes.

Table 12
A comparison of the average MSEs for different methods on forecasting the exchange rates.

	Methods		Average MSEs
One-day forecasting	Random walk (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.011100
	Radial basis function neural network (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.035900
	Leu et al.'s method (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.006500
	Two-factors second-order fuzzy-trend logical relationship groups and PSO techniques [49]	Use PY/USD	0.004954
		Use RW/USD	0.005100
		Use NY/USD	0.004961
		Use TAIEX	0.004869
		Average	0.004971
	The proposed method		**0.004670**
Three-day forecasting	Random walk (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.037600
	Radial basis function neural network (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.077200
	Leu et al.'s method (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.028300
	Two-factors second-order fuzzy-trend logical relationship groups and PSO techniques [49]	Use PY/USD	0.005668
		Use RW/USD	0.004963
		Use NY/USD	0.004727
		Use TAIEX	0.004759
		Average	0.005029
	The proposed method		**0.004664**
Five-day forecasting	Random walk (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.058200
	Radial basis function neural network (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.087000
	Leu et al.'s method (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.050100
	Two-factors second-order fuzzy-trend logical relationship groups and PSO techniques [49]	Use PY/USD	0.004986
		Use RW/USD	0.004967
		Use NY/USD	0.004954
		Use TAIEX	0.005493
		Average	0.005100
	The proposed method		**0.004666**
Seven-day forecasting	Random walk (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.101000
	Radial basis function neural network (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.065000
	Leu et al.'s method (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]		0.053200
	Two-factors second-order fuzzy-trend logical relationship groups and PSO techniques [49]	Use PY/USD	0.005675
		Use KRW/USD	0.005758
		Use CNY/USD	0.005936
		Use TAIEX	0.005979
		Average	0.005837
	The proposed method		**0.004665**

smoothing method is used to filter the noise in the time series. By using hybrid multi-order fuzzy time series, our model is proved to be more stable and efficient, and with different evaluation standards, it performs well on different stock indexes. Compared with different fuzzy time series prediction models, our experiments demonstrate that the proposed model significantly outperforms other models and has excellent generalization ability.

Our contributions are shown as follows:

Table 13
A comparison of the average MSEs of the four predicted values for different methods in Table 12.

Methods	Average MSEs
Random walk (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]	0.051975
Radial basis function neural network (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]	0.066275
Leu et al.'s method (use the combination of JPY/USD,KRW/USD,CNY/USD and TAIEX) [48]	0.034525
Two-factors second-order fuzzy-trend logical relationship groups and PSO techniques [49]	0.005234
The proposed method	**0.004666**

1. The directional accuracy rate (DAR) of a forecasting model is selected as one way of performance measures. As far as we know, this is the first work that measures the performance of different models according to DAR and RMSE.
2. Exponential smoothing is used to eliminate noise in the time series.
3. Multi-order and multivariate fuzzy time series are combined to forecast financial time series.
4. The proposed method is a novel forecasting model and performs very well for different financial time series.

The future work is to find a better partition method of the optimal fuzzy subset and to apply the proposed method for solving other forecasting problems.

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